DOCUMENT RESUME

ED 207 848 SE 035 719

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TITLE Distance University Students' Use of Course Material to Complete Mathematics Exercises.
PUB DATE Aug 81
NOTE 23p.; Paper presented at the Forum of the Australian and South Pacific External Studies Association (Suva, Fiji, August, 1981).

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Cognitive Processes; *College Mathematics; Educational Research; Higher Education; Individualized Instruction; Mathematics Instruction; Mathematics Materials; *Problem Solving; *Undergraduate Study
IDENTIFIERS *Heuristic Methods; *Mathematics Education Research

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This study reports an investigation of the way in which distance students use course materials and plan their work in order to complete mathematics exercises. Two undergraduate students of mathematics and one graduate student were asked to complete a course exercise using standard course materials under distance studying conditions. They verbalized their thoughts while working. The verbal protocols of students' problem-solving processes and use of course materials were described and compared using 12 component procedures of mathematics problem-solving. It was found that the two undergraduates centered on single aspects of the problem. They used examples rather than conceptual information presented in the written materials. The graduate made greater use of theories and definitions, but like the undergraduates, neglected the expository material. No student planned the work. The data and a review of the problem-solving literature indicate the need for course designers and tutors teaching mathematics via written materials to know how those materials are interpreted and used by their students. (Author)

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DISTANCE UNIVERSITY STUDENTS' USE OF COURSE MATERIAL TO COMPLETE MATHEMATICS EXERCISES

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ABSTRACT

This study reports an investigation of the way in which distance students use course materials and plan their work in order to complete mathematics exercises. Two undergraduate students of mathematics and one graduate were asked to complete a course exercise using standard course materials under distance studying conditions. They verbalized their thoughts while working.

The verbal protocols of students' problem-solving processes and use of course materials were described and compared using twelve component procedures of mathematics problem-solving.

It was found that the two undergraduates centred on single aspects of the problem. They used examples rather than conceptual information presented in the written materials. The graduate made greater use of theories and definitions, but like the undergraduates, neglected the expository material. No student planned the work. The data and a review of the problem-solving literature indicate the need for course designers and tutors teaching mathematics via written materials to know how those materials are interpreted and used by their students.

When a student learns introductory university mathematics at a distance from campus, s/he is greatly dependent on packages of course materials, and on tutor feed-back to worked exercises for information about his/her own performance. The tutor obtains information about the student's performance from the way exercises are solved, and from personal communication. Exercises are usually designed to assess how well a distant student has understood and applied the mathematical concepts and skills described in the course materials.

Materials used by distant students usually consist of prescribed textbooks and reference works, written course notes and any study aids provided by the university. Ideally these materials do not only contain the necessary mathematical information, but also are designed to help the student understand that information, and to develop the mathematical skills by which information can be applied to set exercises.

Course materials need to contain information of a primary nature such as definitions, theorems and proofs, and expositions and illustrations of the application of this primary information to problems and examples. Exercises and assignments should provide opportunities for the distant student to explore and assimilate the concepts, and to develop problem-solving skills and strategies suitable for working mathematical computations. The completed exercises should indicate to the tutor how well a student is grasping information and developing mathematical techniques.

However, written and completed exercises may yield only limited information about a student's understanding. The presentation of a neat, final draft may not reflect the processes by which the exercise was interpreted, and relevant knowledge was annexed and applied to the question. The difficulties which the student encountered in the course

of completing the exercise may not be in evidence, since misapprehensions and errors may not be revealed in the finished work. The campus tutor may have no way of effectively diagnosing difficulties and insufficiencies in the students' knowledge and skills, and of correcting mistakes before they become part of the students' learned behaviour.

Mathematics teaching in schools has been advanced by the use of process analyses of the way students work through exercises and problems. Descriptions of the procedures by which children solve particular problems has allowed teachers to diagnose inadequate knowledge and difficulties in applying that knowledge. Detailed diagnosis of errors provides effective targets for specific interventions and remedial teaching (Newman, 1977; Casey, 1978; Hollander, 1978; Radatz, 1979; Clements, 1980).

It seemed reasonable to assume that a process analysis of university students' working of exercises would yield information about their working of mathematical exercises, and their associated use of written materials.

Very little is known about how distant students actually study. Most of the existing data refers to enrolment patterns and success or failure rates, or institutional records (Dodds, Guiton & Lawrence, 1981). In particular the way distant students use course materials has only recently received any empirical scrutiny. Bates (1976) reported that British Open University mathematics students tended to listen to fewer of the available broadcast lessons than students in other faculties. Burt (1976) reported that of ten mathematics students, five indicated that they completed their assessment questions while they read the course text and five did so after they completed their reading.

Dodds & Lawrence (1979) described how three adult students of educational psychology used their course materials for a different purpose from that intended by the course designer. The course had been prepared as an integrated package of knowledge and skills. The students had used the

written materials as a smorgasbord from which they sampled information that suited their own current interests and needs, such as information they could use in their school teaching. The study used problem-solving paradigms such as "think aloud" verbalizations to obtain data of the students' studying behaviour.

A similar problem-solving approach permits the examination of how a student actually interprets an exercise, uses the course materials, and applies mathematical knowledge to the solution of a particular exercise.

Studies of adult problem-solving have yielded information about people's knowledge and skill in the solution of exercises in the domains of chess (Chase & Simon, 1973), logic (Newell & Simon, 1972) and mathematics (Polya, 1954; 1957, Gagné & Smith, 1962). The researcher usually asks a person to solve an exercise while "thinking out loud" and audiotapes the person's verbalizations. Although the verbalized reports may not yield complete parallels of mental processes (Gagné & Smith, 1962) the technique provides unique, rich and analyzable information about a problem-solver's method of tackling a task (Newell & Simon, 1972; Ericsson, 1975; Lawrence, 1979(b)).

The purpose of the present study was (1) to develop a descriptive device for examining how mathematics students worked on an exercise, and (2) to use that device to describe differences in the problem-solving procedures and use of course materials by two undergraduate students and one graduate of mathematics courses.

Using pilot data the descriptive device was developed as a taxonomy to (1) describe the procedures which subjects used when solving a mathematics exercise, and to (2) provide an empirical basis for classifying the statements made by a particular subject when he or she thought aloud while attempting to solve a mathematics exercise.

This taxonomy was generated from a set of propositions about the nature of problem-solving and from verbal protocols obtained during the pilot study. The propositions were mainly developed from a study of the sub-processes of adult problem-solving proposed by Polya (1957), Newell & Simon (1972), and Pitt (1976). Polya's description of four sub-processes used by people when they solve mathematics problems was one of the first suggestions that people use specific processes when solving a problem, and that these processes can be sequenced. Newell & Simon's General Problem Solver was developed from a detailed study of verbal protocols taken while students attempted to solve logic problems. Newell & Simon showed that sub-goaling is a process which people use when they solve a complex problem. Pitt developed a comprehensive model of problem-solving to describe how expert chemists, and college and high school students solve chemistry problems. Her model combines Polya's emphasis on definition of the problem, and Newell & Simon's concept of sub-goaling. It showed how her different subject groups' solutions could be described by a set of discrete procedures. She showed that the experts used procedures to define the problem and to formulate and test hypotheses more consistently than the college students, who used these procedures more frequently than the high school students. The basic concepts of the three models were used to generate a set of twelve component procedures which would be used to describe the thinking aloud reports obtained from the undergraduate students while they solved a mathematics exercise related to a section of a current mathematics course. A graduate, who had completed the course previously , was used as a referent subject. It was expected that the graduate student's greater experience in studying mathematics would yield differences in the way she tackled the exercise and in her use of course materials to solve an undergraduate course exercise.

The Descriptive Taxonomy

The taxonomy is comprised of 12 component procedures for the solution of a mathematics exercise. The taxonomy is shown in Figure 1. Seven procedures (1-6, 10) are dependent on the first two steps of Polya's (1957) heuristic related to the importance of planning. Three others (7, 8, 11) have similarities with Newell & Simon's concept of subgoaling. Together, these procedures are similar to Pitt's subroutines related to the definition of a problem and the acquisition of data. Two component procedures (9, 12) are dependent on Polya's fourth sub-process of examining the solution. The procedures are grouped in four major phases of problem-solving following Polya's model. The phases are: (I) Assembling the Given Data, (II) Planning the Solution, (III) Executing the Solution, and (IV) Checking the Solution.

The component procedures were used to describe each separate proposition which a student verbalized when working on the exercise. Each student's complete verbal transcript was subdivided into verbal units, with each verbal unit expressing one complete thought, idea or operation (Lawrence, 1979(a)). One component procedure was used to describe each of these verbal units. The labelled component procedures were treated as nominal categories for subsequent data analysis.

FIGURE 1 ABOUT HERE

Method

Subjects

Two of the three subjects of the study were undergraduate students taking a first year university mathematics course by a distance mode of study. One undergraduate was a male (U1) and one a female (U2). The third subject was a female mathematics graduate who had undertaken a mathematics course by a distance mode of study from the same university two years previously and was currently enrolled in a different campus-based course (G3). All students lived within 20 kilometres of the university and were selected for the level of their mathematics studies, their present or previous enrolment in a mathematics course as a distant student, and their availability for interview.

Instruments and Task

1. Students were asked to complete an exercise which had been set for students enrolled in the on-campus mode of the same mathematics course. The exercise was related to the section of the course on limit theory, and was presented to the students in written form on a 76mm x 127 mm card as follows:

   "Show directly from the definition that

   $$\lim_{x \to 1} \frac{x}{x + 1} = \tfrac{1}{2}$$"

2. The undergraduate students used their own copies of the prescribed course text (Swokowski, 1975); the graduate student used her own copy of a similar course text with which she was familiar (Purcell, 1976). Both books covered the content necessary to complete the exercise. Each student was supplied with a copy of the course notes.

Procedure

The students were interviewed individually. Subjects U1 and G3 were interviewed at home and Subject U2 in an office at the University. Each student was presented with the exercise and instructed to think aloud as he or she attempted to solve it. Instructions were given to use the text and course notes freely, and to use the provided worksheets. Students' verbalizations were audiotaped. Each interview was of approximately 30 minutes duration. The experimenter maintained an observation schedule to coordinate the audiotaped verbalizations with the behavioural observations. Standard probes were used when a student failed to make any verbalization during a reasonable interval, and when a student ceased verbalizing while reading or writing.

The audiotape was transcribed for analysis. The three students' verbal reports of their problem-solving operations were divided into verbal units. Each unit was described by one of the 12 component procedures of the taxonomy shown in Figure 1.

Results

The data are reported as (1) students' comparative use of the 12 component procedures, (2) students' comparative use of three component procedures on course materials, and (3) each student's profile of use of the 12 component procedures.

1. Students' Comparative Use of 12 Component Procedures

Each student's use of a particular component procedure was expressed as a proportion of that student's total number of individual verbal units. The total use of all component procedures by the three students is shown in Table 1. For example Column 1 shows that student G1's use of the component procedure (1) Assembling of the given data represented .21 of her 153 separate verbal units.

The frequency of expression of each component procedure was compared using a one-way Chi-square test. For procedures in which the three students' frequency of expression was significantly different, pairs of proportions were compared using Ferguson's test for the significance of a difference between independent proportions. This test yields a z-score. (Ferguson, 1956, p.146).

Insert Table 1 about here

(a) The graduate student expressed more individual verbal units (153) than either of the undergraduates while working on the exercise. U2 expressed 133 and U3 47.

(b) The three students' frequency of expression differed significantly for seven of the 12 component procedures. The graduate student used two of them significantly more frequently than at least one undergraduate. The following differences were obtained:

i. The graduate (G1) used the procedure "Looking up a Definition" significantly more frequently than both the undergraduates, $\chi^2(2) = 14.04$, $p<.001$.

ii. The graduate (G1) used the procedure "Assembling Internal Given Data" significantly more frequently than only one undergraduate (U2), $z = 2.56$, $p<.01$.

iii. The graduate (G1) used the procedure "Executing a Subgoal" significantly more frequently than the other undergraduate (U3), $z = 3.15$, $p<.01$.

iv. Undergraduate (U2) used the procedure "Executing a Subgoal" significantly more frequently than both other students, $\chi^2(2) = 14.47$, $p<.001$.

v. Undergraduate (U3) used the procedure "Looking up an Illustration" significantly more frequently than both other students, $\chi^2(2) = 103.1$, $p<.001$.

2. Students' Comparative Use of the Three Procedures Relating to Course Materials

The three students' relative use of the three procedures related to course materials (procedures 2, 3, and 4) was compared. A students use of a component procedure was expressed as a proportion of the total number of that student's verbal units which related to course material. The three students' comparative use of the three procedures related to course materials is shown in Table 2, and illustrated in Figure 2. For example, Table 2, Column 1 shows that graduate G1's use of the procedure "Looking up a Definition" accounted for .71 of that student's 45 verbal units relating to her use of course materials. Frequency of expression of each of the three procedures were compared using Ferguson's test for the significance of a difference between independent proportions.

Insert Table 2 about here

Insert Figure 2 about here

The graduate expressed 45 verbal units associated with her use of course materials, undergraduate U2 expressed 33, and undergraduate U3 expressed 41. These represent .29, .25 and .87 respectively of those students' total number of individual verbal units.

The three students' frequency of expression differed significantly for all three component procedures. One undergraduate also used one of the three procedures significantly more frequently than the other undergraduate.

i. The graduate (G1) used the procedure "Looking up a Definition" significantly more frequently than both undergraduate students, $\chi^2(2) = 25.95$, $p<.001$.

ii. Undergraduate (U2) was the only subject to use the procedure "Looking up an Exposition".

iii. Both undergraduates used "Looking up an Illustration" more frequently than the graduate.

iv. Undergraduate (U3) used the procedure "Looking up an Illustration" significantly more frequently than both other students, $\chi^2(2) = 16.57$, $p<.001$.

v. Of the two undergraduates, U2 used "Looking up a Definition" significantly more frequently than U3, $z = 2.03$, $p<.05$.

3. Students' Individual Use of the 12 Component Procedures

1. The graduate G1 expressed use of nine component procedures. Three of these accounted for over two thirds of her verbal units: "Assembling internal given data" (.21), "Looking up a definition" (.21), and "Executing a subgoal" (.25). She did not express procedures for looking up expositions or developing plans. Student G1's proportional expression of the 12 procedures is illustrated in Figure 3(a).

Insert Figure 3 about here

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2. Undergraduate U2 expressed use of 10 component procedures. One of these, "Executing a subgoal" was expressed more frequently than any other procedure (.38). The nine other

procedures expressed by her each accounted for less than 15% of her verbal units. She did not express a procedure for developing plans. Undergraduate U2's proportional expressions of the 12 procedures is illustrated in Figure 3(b).

3. Eighty-one percent of Undergraduate U3's verbal units were devoted to "Looking up an illustration". Only three other procedures were used. He did not express procedures for looking up expositions, developing plans, identifying sub-goals or checking the solution. Undergraduate U2's proportional expression of the 12 procedures is illustrated in Figure 3(c).

In summary, all three students failed to solve the exercise, and all three demonstrated minimal or no use of planning and checking strategies in their work.

All three students differed in the number and frequency of expression of procedures used to solve the exercise. They also differed in the frequency of expression of procedures related to the use of course materials. The graduate expressed more individual verbal units than the undergraduates and showed greater use of given information and mathematical definitions than either of the undergraduates.

The undergraduate students tended to concentrate on a single procedure and to focus on the worked examples in the course materials. One undergraduate (U2) relied heavily on the execution of sub-goals, and the other (U3) relied almost exclusively on looking up illustrations. The graduate was more even in her use of the component procedures. The graduate used course materials to look up definitions while the undergraduates used them to look up illustrations.

Discussion

The description of the student's verbalized working of the exercise was able to show how they were tackling the task, their concentration on different mathematical information, and their use of course materials to solve the exercise. This type of information would not be readily available from a final draft of a worked exercise. For example without such a detailed analysis of each student's working procedures, it would not be possible to discover that the undergraduates used the course materials chiefly as a source of type examples which could be transformed to find the solution of the exercise. Their studying behaviour which disregarded relevant definitions and made minimal use of given data is the kind of information which would useful for a tutor to know. Their lack of planning which Polya has shown to be inefficient was able to be diagnosed using the taxonomy. These three aspects of the students' behaviour would provide a tutor with valuable insight into their studying patterns.

The data indicate the instructional usefulness of obtaining detailed information about the way a student works on a task.

The information obtained in this study needs to be interpreted in relation to the sample's size, and method of selection, and relation to the particular exercise used as a task. However the wide-spread use of written materials in external tuition demands the collection of information about the way these materials are used, and the effect of those materials on student learning and performance. We have shown that it is possible to show how students use course materials in a problem-solving analysis of their expressed work on one exercise. The technique has provided additional information about how the students approached an educational task. Extension of the approach and replication with other exercises and different features of written course materials give promise of useful data for designers of courses.

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Table 1

Three students' Proportional Use of the Twelve Component Procedures

Component Procedure	Subject G1	Subject U2	Subject U3
Phase I: Assembling Given Data			
1. Assembling internal given data*	.21	.10	.09
2. Looking up a definition*	.21	.06	.06
3. Looking up an exposition*	.00	.07	.00
4. Looking up an illustration*	.09	.11	.81
5. Locating given data elsewhere	.01	.00	.00
Phase II: Planning			
6. Developing a plan	.00	.00	.00
7. Identifying a subgoal	.14	.07	.00
Phase III: Executing			
8. Executing a subgoal*	.25	.38	.04
Phase IV: Checking			
9. Checking goal achieved	.01	.04	.00
Expressed Inabilities			
10. Inability to develop a plan	.03	.03	.00
11. Inability to execute a subgoal	.05	.08	.00
12. Inability to achieve a goal	.00	.06	.00
	1.00	1.00	1.00
Total Number of Verbal Units	153	133	47
Average Number of Verbal Units Per Minute	7.29	8.80	2.92

* Chi-square comparison significant, $p<.01$.

Table 2

Three students' Proportional Use of Course Materials

Component Procedure	Subject G1	U2	U3
2. Looking up a definition*	.71	.24	.07
3. Looking up an exposition*	.00	.30	.00
4. Looking up an illustration*	.29	.46	.93
Total Number	1.00	1.00	1.00
Number of Verbal Units Used on Use of Course Materials	45	33	41

* Chi-square comparison significant, $p<.001$.

PHASE I: ASSEMBLING GIVEN DATA	
1. Assembling Internal Given Data	a goal, rule or condition is identified without reference to any external source.
2. Looking up a Definition	a definition, theorem, axiom, rule is identified while referring to a printed source.
3. Looking up an Exposition	an explanation or interpretation of a definition is identified while referring to a printed source.
4. Looking up an Illustration	a printed example or application is identified while referring to a printed source.
5. Locating Given Data Elsewhere	a goal, rule, or condition to be followed identified while referring to a source which is not printed.
PHASE II: PLANNING	
6. Developing a Plan	a completely or partially constructed solution in general terms.
7. Identifying a Subgoal	discerning one step which needs to be taken or two items of data which need to be related.
PHASE III: EXECUTING	
8. Executing a Subgoal	finding a relationship between two items of data, or performing a single step of a solution.
PHASE IV: CHECKING	
9. Checking Goal Achieved	asking if the goal of the exercise has been achieved, the correct rules applied or the given conditions adhered to.
EXPRESSED INABILITIES	
10. Inability to Develop a Plan	failure to completely or partially construct a solution in general terms.
11. Inability to Execute a Subgoal	failure to find a relationship between two items of data or to execute a single step of a solution.
12. Inability to Achieve a Goal	failure to achieve the goal of the exercise, apply the correct rules or adhere to the given conditions.

Figure 1: Taxonomy of Component Procedures of Mathematics Exercise Solution Grouped in Four Phases.

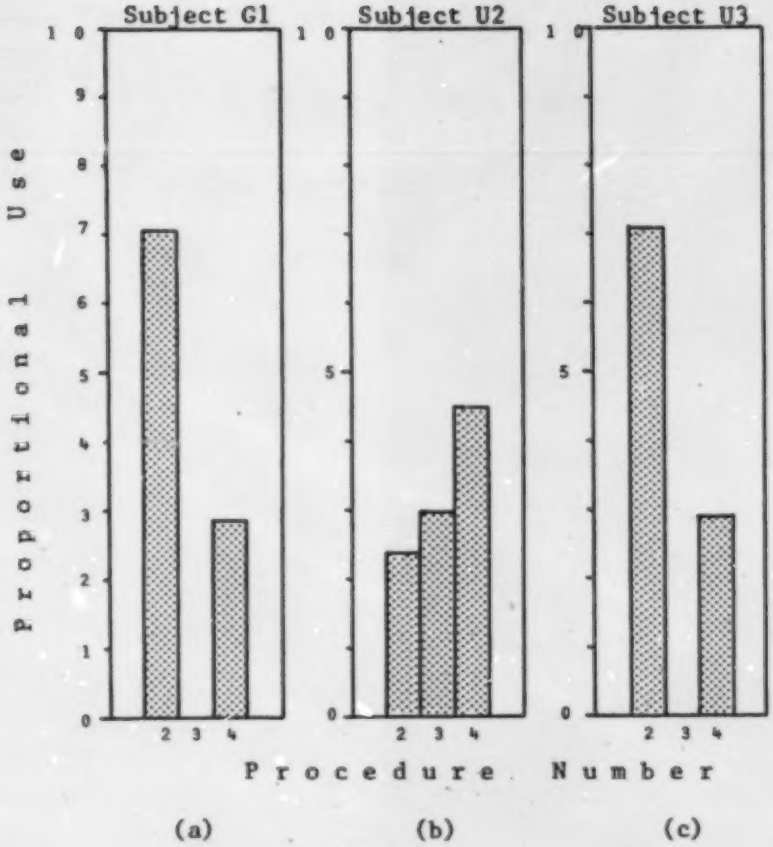

Figure 2: Three students' proportional use of course materials.

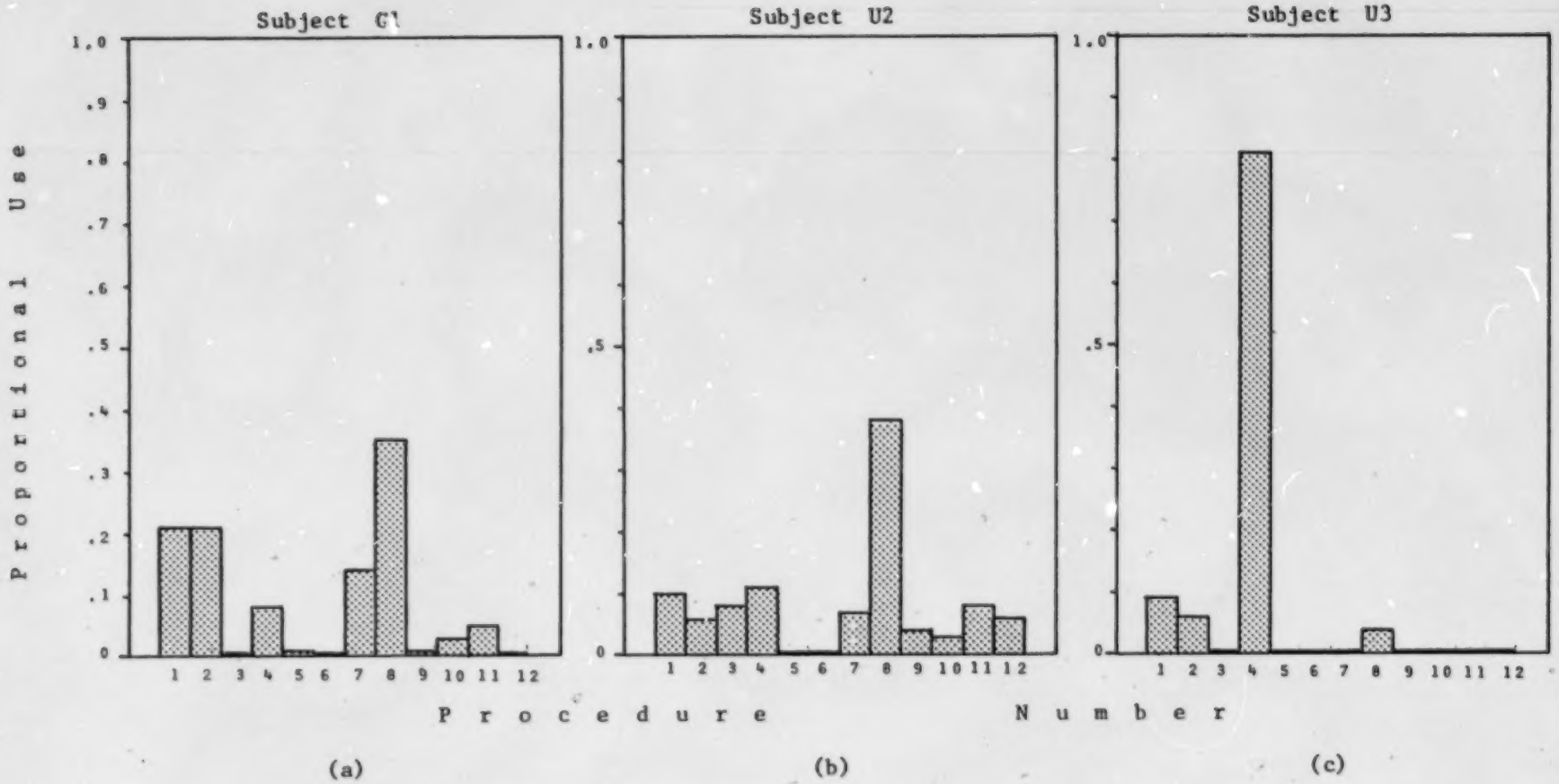

Figure 3: Three students' proportional use of the twelve component procedures.